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Towards covariant quantization of superstrings without pure spinor constraints

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Abstract: We construct a covariant quantum superstring, extending Berkovits' approach by introducing new ghosts to relax the pure spinor constraints. The central charge of the underlying Kac-Moody algebra, which would lead to an anomaly in the BRST charge, is treated as a new generator with a new $b - c$ system. We construct a nilpotent BRST current, an anomalous ghost current and an anomaly-free energy-momentum tensor. For open superstrings, we find the correct massless spectrum. In addition, we construct a Lorentz invariant B-field to be used for the computation of the integrated vertex operators and amplitudes. We conclude with some remarks of the outstanding and open problems and further developements.

Keywords: Superstrings and Heterotic Strings, Conformal Field Models in String Theory, Space-Time Symmetries, BRST Symmetry.

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1. Introduction and Summary

Recently, a new formulation of superstrings was developed which is explicitly super-Poincaré invariant in $9+1$ dimensions [1]–[4]. It is based on a *free* conformal field theory on the world-sheet and a nilpotent BRST charge Q which defines the physical vertices as representatives of its cohomology. In addition to the conventional variables x^m and θ^α of the Green-Schwarz formalism, a conjugate momentum p_α for θ^α and a set of commuting ghost fields λ^α are introduced. The latter are complex Weyl spinors satisfying the pure spinor conditions $\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta = 0$ [1, 5]. This equation can be solved by decomposing λ with respect to a non-compact U(5) subgroup of SO(9, 1) into a singlet $\underline{1}$, a vector $\underline{5}$, and a tensor $\underline{10}$. The vector can be expressed in terms of the singlet and tensor, hence there are 11 independent complex variables in λ^α.

The pure spinors λ^α are needed to cancel the central charge of the conformal algebra (+10 from the bosonic coordinates x^m, −32 from the spinor variables p_α and θ^α, and +22 from the pure spinors λ^α), to obtain the correct double poles in the Lorentz algebra and, last but not least, to render the operator Q nilpotent.

Of course, the problem of the covariant quantization of superstrings is one of the fundamental problems in string theory. The subject has a long history. We give in [6] some of the early papers on the subject which we used, but the list is far from complete.

As shown by Berkovits in [3], the cohomology of the BRST charge contains exactly the physical spectrum of the superstring and, in particular for the massless states, it provides the covariant equations of motion, those of super Yang-Mills theory in 10 dimensions for the open superstring or those of $N = 2$ supergravity for closed superstrings. For massive states he could not use the formalism based on the U(5)-like subgroup; it was possible to use an SO(8) subgroup, but this introduced an infinite set of ghost-for-ghosts although the cohomology did not depend on these fields.

Although Berkovits's approach provides a way to by-pass many of the difficulties of the Green-Schwarz formalism for a super-Poincaré covariant description of superstrings, an explicit parametrization of pure spinors λ^α is needed at several points in his construction. Using such an explicit parametrization for pure spinors, he was able to define Lorentz currents which satisfy the covariant Lorentz algebra [8]. However, the solution of the pure spinor constraints breaks the explicit covariance and some expressions cannot be written in a covariant way. For example, the action is not covariant and also intermediate steps in the computation of amplitudes are clearly affected. In addition, a conjugate momentum for the pure spinors can only be constructed in an explicit parametrization, resulting in OPE's which are not manifestly covariant. The tree level amplitudes are manifestly super-Poincaré invariant only for external massless states and the pure spinor constraint makes the theory unsuitable for a path integral approach.

The cohomology of [1]–[4] is a constrained cohomology. We therefore decided to try to relax the pure spinor conditions and construct a new BRST operator such that its unconstrained cohomology coincides with the constrained cohomology of Berkovits's approach. The extension of the BRST symmetry to unconstrained λ^α led us to enlarge the field space by adding more ghost fields: an anticommuting vector ξ^m, a commuting spinor χ_α, an anticommuting vector 1-form ω_z^m, and their corresponding antighosts, and further an anticommuting $b - c_z$ system with conformal weights 0 and 1, respectively.

We do obtain a BRST invariant action and the complete physical spectrum, but the present formulation is presumably not the final one. In fact, in subsequent works we found new ingredients which, for example, permit to construct easily the BRST cohomology, even we have also not succeeded in completely elucidating the geometrical meaning of all these new ghosts.

Our final action for the left-moving sector is given by

$$S = \int d^2z \Big(\frac{1}{2}\partial x^m \bar\partial x_m + p_\alpha \bar\partial\theta^\alpha + \beta_{zm}\bar\partial\xi^m + \beta_{z\alpha}\bar\partial\lambda^\alpha + \kappa_z^\alpha\bar\partial\chi_\alpha + c_z\,\bar\partial b + \omega_z^m\bar\partial\eta_m\Big)\,. \quad (1.1)$$

Our final BRST current is given by

$$\begin{aligned} j_z^B &= \lambda^\alpha d_{z\alpha} - \xi^m\Pi_{zm} - \chi_\alpha\partial_z\theta^\alpha - \xi^m\kappa_z^\alpha\gamma_{m\alpha\beta}\lambda^\beta - \frac{1}{2}\lambda^\alpha\gamma^m_{\alpha\beta}\lambda^\beta\beta_{zm} + \\ &\quad + c_z - \frac{1}{2}b\left(\xi^m\partial_z\xi_m - \frac{3}{2}\chi_\alpha\partial_z\lambda^\alpha + \frac{1}{2}\partial_z\chi_\alpha\lambda^\alpha\right) - \frac{1}{2}\partial_z\left(b\,\chi_\alpha\lambda^\alpha\right), \end{aligned} \quad (1.2)$$

and the energy-momentum tensor is

$$T_{zz} = -\frac{1}{2}\Pi_z^m\Pi_{mz} - d_{z\alpha}\partial_z\theta^\alpha - \beta_{zm}\partial_z\xi^m - \beta_{z\alpha}\partial_z\lambda^\alpha - \kappa_z^\alpha\partial_z\chi_\alpha + \partial_z b\,c_z + \partial_z\eta_m\,\omega_z^m\,. \quad (1.3)$$

Because $-(1/2)\Pi_z^m\Pi_{mz} - d_{z\alpha}\partial_z\theta^\alpha = -(1/2)\partial_z x^m\partial_z x_m - p_{z\alpha}\partial_z\theta^\alpha$ we are dealing with a free conformal field theory. The ghost current we use is given by

$$J_z^{gh} = -\left(\beta_{mz}\xi^m + \kappa_z^\alpha\chi_\alpha + \beta_{z\alpha}\lambda^\alpha + b\,c_z + \eta^m\omega_{zm}\right)\,. \quad (1.4)$$

There is also a composite field B_{zz} which will be discussed in the text.

Our main results are:

1. We keep manifest SO(9,1) invariance at every step. No pure spinor constraints or U(5) decompositions appear. As a consequence we can take λ^α to be real which solves one of the puzzles of [1] where λ^α had to be complex in order that the pure spinor constraint have a solution, but where the real θ^α transforms into complex λ^α.

2. We construct a BRST operator with the properties one would like it to have: its current is nilpotent and a primary field. Our BRST charge is based on a Kac-Moody algebra[1] for spacetime symmetries, and it describes the geometry of a $D = (9,1)$ super-space. In the RNS approach the BRST charge describes the dynamics since it starts with $\oint c^z T_{zz} + \cdots$.

3. We obtain a stress tensor T_{zz} which is a good conformal field and whose central charge vanishes.

4. We also construct a composite operator B_{zz} which satisfies the fundamental relation $T_{zz} = [Q, B_{zz}]$ where T_{zz} is given in (1.3). We believe that $\oint B_{zz}$ plays the same role as the zero mode b_0 in the RNS approach. The integrated vertices $\oint dz \mathcal{V}_z^{(0)}$ are constructed from the unintegrated vertices $\mathcal{U}^{(1)}$ by $\mathcal{V}_z^{(0)} = \oint dz B_{zz} \mathcal{U}^{(1)} + [Q, X_z]$.

5. We require that the unintegrated vertex operators should satisfy $[Q, \mathcal{U}^{(1)}] = 0$. This selects all possible deformations of the Kac-Moody algebra, in other words, the BRST charge, with Kac-Moody generators replaced by Kac-Moody generators plus connections, is still nilpotent. To be physical observables these deformations must be primary spin-1 fields, so $\mathcal{V}_z^{(0)}$ should have conformal spin 1. This second constraint brings in the dynamics. For the massless sector of the open superstring, the first constraint, namely $[Q, \mathcal{U}^{(1)}] = 0$, implies that the deformations are described by a super gauge-connection A_α, A_m and field strength A^α (usually denoted by W^α) which satisfies the proper Bianchi identities. The second constraint implies the equations of motion for super Yang-Mills theory (at the linearized level). The conditions $F_{\alpha\beta} = F_{\alpha m} = 0$ for the superspace curvature of the gauge connection follow as a consequence of the equations of motion.

The paper is organized in the following way: in section 2, we discuss the construction of the BRST charge based on an extension of Berkovits's approach and promote the central charge of the Kac-Moody algebra to an operator. In section 3, we prove the BRST invariance of the action and derive the boundary conditions for the open superstring needed to maintain BRST invariance and supersymmetry at the boundaries. Section 4 is devoted to the construction of the energy-momentum tensor, the field B_{zz}, the ghost current and to compute their OPE's. In section 5, we discuss briefly the relation with the RNS approach and, finally, in section 6 we study the massless sector of open string. Section 7 contains some comments and speculations.

[1]These algebras were discovered in physics in ref. [7] where they were called affine Lie algebras.

The present article was completed in the fall 2001, but since then we have made substantial progress in answering some of the questions left open in this article. We also received useful comments from the referee. We added at the end of this article a new section in which we give more justification for our approach. We have decided to keep the original presentation unchanged, because it offers in our opinion the clearest introduction to our work.

2. The BRST charge

On a flat worldsheet, the left- and right-moving contributions to the classical covariant Green-Schwarz superstring action [9] without cross interactions can be written as

$$S = \int d^2 z \left(\frac{1}{2} \partial x^m \overline{\partial} x_m + p_\alpha \overline{\partial} \theta^\alpha + \hat{p}_\alpha \partial \hat{\theta}^\alpha \right) , \tag{2.1}$$

where $\partial_z = (1/2)(\partial_\sigma - i\,\partial_\tau)$ and $\bar{\partial}_z = (1/2)(\partial_\sigma + i\,\partial_\tau)$. Furthermore, p_α (or its anti-holomorphic partner $\hat{p}_\alpha$) is related to x^m and θ^α by the constraint $d_\alpha = 0$ with [10]

$$d_\alpha = p_\alpha - \frac{1}{2} \partial x_m \gamma^m_{\alpha\beta} \theta^\beta - \frac{1}{8} \gamma^m_{\alpha\beta} \theta^\beta \gamma_{m\,\gamma\delta} \theta^\beta \theta^\gamma \partial \theta^\delta \, . \tag{2.2}$$

Setting d_α equal to zero, (2.1) leads to an interacting superstring whose left-movers do not interact with its right-movers, and which therefore is not equal to the Green-Schwarz superstring. The variables x^m and θ^α are worldsheet scalar fields. On the other hand, the conjugate momenta p_α, d_α and $\Pi^m = \partial x^m + (1/2)\theta^\alpha \gamma^m_{\alpha\beta} \partial \theta^\beta$ carry a world-sheet vector index, i.e. $p_{z,\alpha}, d_{z,\alpha}, \Pi_{z,m}$. The index z will be omitted in the following when there is no ambiguity. The symbols $\gamma^m_{\alpha\beta}$ and $\gamma^{m\,\alpha\beta}$ are real 16×16 symmetric matrices which are the off-diagonal elements of the 32×32 Dirac-matrices[2] and which satisfy $\gamma^m_{\alpha\beta}\gamma^{n\,\beta\gamma} + \gamma^n_{\alpha\beta}\gamma^{m\,\beta\gamma} = 2\eta^{mn}\delta^\gamma_\alpha$ and $\gamma_{m\,(\alpha\beta}\gamma^m_{\gamma)\delta} = 0$. The latter relation makes Fierz rearrangements very easy.

Furthermore, we have [10]

$$\begin{aligned} d_\alpha(z) d_\beta(w) &\sim -\frac{\gamma^m_{\alpha\beta}\Pi_m(w)}{z-w} , & d_\alpha(z)\Pi^m(w) &\sim \frac{\gamma^m_{\alpha\beta}\partial\theta^\beta(w)}{z-w} , \\ \Pi^m(z)\Pi^n(w) &\sim -\frac{1}{(z-w)^2}\eta^{mn} , & d_\alpha(z)\theta^\beta(w) &\sim \frac{1}{z-w}\delta_\alpha{}^\beta , \end{aligned} \tag{2.4}$$

[2] One may use ten real $D = (9,1)$ Dirac-matrices $\Gamma^m = \{I \otimes (i\tau_2), \sigma^\mu \otimes \tau_1, \chi \otimes \tau_1\}$ where $m = 0, \ldots, 9$ and $\mu = 1, \ldots, 8$. The σ^μ are eight real symmetric 16×16 off-diagonal Dirac matrices for $D = (8,0)$, while χ is the real 16×16 diagonal chirality matrix in $D = 8$ [11]. The chirality matrix in $D = (9,1)$ is then $I \otimes \tau_3$ and the $D = (9,1)$ charge conjugation matrix C, satisfying $C\,\Gamma^m = -\Gamma^{m,T} C$, is numerically equal to $C = \Gamma^0$. If one uses spinors $\Psi^T = (\alpha_L, \beta_R)$ with spinor indices α^α_L and $\beta_{R,\dot\beta}$, the index structure of the Dirac matrices and the charge conjugation matrix is

$$\Gamma^m = \begin{pmatrix} 0 & (\sigma^m)^{\alpha\dot\beta} \\ (\tilde\sigma^m)_{\dot\beta\gamma} & 0 \end{pmatrix} , \qquad C = \begin{pmatrix} 0 & c_\alpha{}^{\dot\beta} \\ c^{\dot\beta}{}_\gamma & 0 \end{pmatrix} , \tag{2.3}$$

where $\sigma^m = \{I, \sigma^\mu, \chi\}$ and $\tilde\sigma^m = \{-I, \sigma^\mu, \chi\}$. The matrices $c_\alpha{}^{\dot\beta}$ and $c^{\dot\beta}{}_\gamma$ are numerically equal to $I_{16\times16}$ and $-I_{16\times16}$, respectively. In the text, we use the symmetric matrices $\gamma^{m\,\dot\alpha\dot\beta} = c^{\dot\alpha}{}_\beta(\sigma^m)^{\beta\dot\beta}$ and $\gamma^m_{\alpha\beta} = c_\alpha{}^{\dot\beta}(\tilde\sigma^m)_{\dot\beta\beta}$, and we omit the dots. The spinors α_L and β_R form inequivalent representations of SO(9,1). We cannot raise and lower the spin indices with the charge conjugation matrix, but $\alpha^\alpha_L c_\alpha{}^{\dot\beta} \beta_{R,\dot\beta}$ is Lorentz invariant.

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where $\sim$ denotes the singular contribution to the OPE's. The operators d_α and Π^m are invariant under the spacetime supersymmetry generated by

$$
\begin{aligned}
Q_\epsilon &= \oint dz\, \epsilon^\alpha j^{su}_{z\alpha}\,, \\
j^{su}_{z\alpha} &\equiv p_\alpha + \frac{1}{2}\partial x^m \gamma^m_{\alpha\beta}\theta^\beta + \frac{1}{24}\gamma^m_{\alpha\beta}\theta^\beta\theta^\gamma\gamma_{m\,\gamma\delta}\partial\theta^\delta\,, \\
j^{\rm su}_{z\alpha}(z) j^{\rm su}_{z\beta}(w) &\sim \frac{\gamma^m_{\alpha\beta}}{(z-w)}\left(\partial x_m + \frac{1}{6}\theta^\gamma\gamma_{m\gamma\delta}\partial\theta^\delta\right)(w) - \frac{1}{3(z-w)^2}(\gamma^m\theta)_\alpha(z)(\gamma_m\theta)_\beta(w)\,,
\end{aligned}
\tag{2.5}
$$

where ϵ^α is a constant Majorana-Weyl spinor. The supersymmetry transformations of the fields are given by

$$
\begin{aligned}
[Q_\epsilon, x^m] &= -\frac{1}{2}\epsilon^\alpha\gamma^m_{\alpha\beta}\theta^\beta\,, \qquad [Q_\epsilon, \theta^\alpha] = \epsilon^\alpha\,, \\
[Q_\epsilon, p_{z\alpha}] &= \frac{1}{2}\partial_z x_m \gamma^m_{\alpha\beta}\epsilon^\beta - \frac{1}{8}\gamma^m_{\alpha\beta}\partial_z\theta^\beta\Big(\epsilon\gamma_m\theta\Big)\,.
\end{aligned}
\tag{2.6}
$$

The susy commutator vanishes on p_α in agreement with the vanishing of the anticommutator of the θ-dependent terms in the r.h.s. of (2.5) with p_α. We require the BRST charge to be susy invariant and therefore it is constructed on the basis $(\Pi^m_z, \partial_z\theta^\alpha, d_{z\alpha})$. All the ghosts and antighosts will be susy inert.

The variables (x^m, θ^α) are the coordinates of $N = 1$ superspace in $D = (9, 1)$ dimensions and $(\Pi^m_z, \partial_z\theta^\alpha, d_{z\alpha})$ form a basis for susy-invariant super 1-forms. At the classical level, a general super 1-form can be written as

$$
\mathcal{V}^{(0)}_z = \Pi^m_z A_m(x,\theta) + \partial_z\theta^\alpha A_\alpha(x,\theta) + d_{z\alpha}A^\alpha(x,\theta)\,,
\tag{2.7}
$$

for the holomorphic sector, or

$$
\begin{aligned}
\mathcal{V}^{(0)}_{z\bar z} = \;& \Pi^m_z\bar\Pi^n_{\bar z}\, G_{m\hat n}(x,\theta,\hat\theta) + \partial_z\theta^\alpha\bar\Pi^n_{\bar z}\, G_{\alpha\hat n}(x,\theta,\hat\theta) + \partial_z\theta^\alpha\partial_{\bar z}\hat\theta^{\hat\alpha}\, G_{\alpha\hat\alpha}(x,\theta,\hat\theta) + \\
& + \Pi^m_z\partial_{\bar z}\hat\theta^{\hat\alpha}\, G_{m\hat\alpha}(x,\theta,\hat\theta) + d_{z\alpha}\bar\Pi^n_{\bar z}\, G^\alpha{}_{\hat n}(x,\theta,\hat\theta) + \Pi^m_z\hat d_{\bar z\hat\alpha}\, G_m{}^{\hat\alpha}(x,\theta,\hat\theta) + \\
& + d_{z\alpha}\partial_{\bar z}\hat\theta^{\hat\alpha}\, G^\alpha{}_{\hat\alpha}(x,\theta,\hat\theta) + \partial_z\theta^\alpha\hat d_{\bar z\hat\alpha}\, G_\alpha{}^{\hat\alpha}(x,\theta,\hat\theta) + d_{z\alpha}\hat d_{\bar z\hat\alpha}\, G^{\alpha\hat\alpha}(x,\theta,\hat\theta)\,.
\end{aligned}
\tag{2.8}
$$

for the closed string sector. These expressions will be useful for the cohomology of the string. For the open string the functions A_m, A_α and A^α are arbitrary superfields of the holomorphic supercoordinates $(x^m_L(z), \theta^\alpha(z))$ and for type-II B strings the $G_{m\hat n}, \ldots, G^{\alpha\hat\alpha}$ are superfields on the superspace defined by $x^m(z,\bar z)$, $\theta^\alpha(z)$, $\theta^{\hat\alpha}(\bar z)$. The fields x^m and θ^α are on-shell, so that we can use conformal field theory techniques. For type-II A strings, we should use $\hat\theta_\alpha$ instead of $\hat\theta^\alpha$.

In addition to the usual ten-dimensional superspace variables x^m and θ^α, Berkovits [1] (see also [4]) introduced a commuting space-time spinor (worldsheet scalar) $\lambda^\alpha(z)$ satisfying the pure spinor condition [12]

$$
\lambda^\alpha\gamma^m_{\alpha\beta}\lambda^\beta = 0\,,
\tag{2.9}
$$

where m runs from 0 to 9. In this approach, the 16 fields λ^α must be complex to allow a solution of (2.9). One may solve (2.9) by decomposing the spinor λ^α with respect

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to a noncompact subgroup of SO(9,1). Introducing 5 creation operators $\Gamma^a = (\Gamma^1 + i\,\Gamma^2)/2, \ldots, (\Gamma^9 + \Gamma^0)/2$ and 5 annihilation operators $\Gamma_a = (\Gamma^1 - i\,\Gamma^2)/2, \ldots, (\Gamma^9 - \Gamma^0)/2$, and defining λ^α to consist of the terms with an even number of these creation operators one obtains $\lambda^\alpha = \lambda_+|0\rangle + (1/2)\Gamma^{ab}\lambda_{ab}|0\rangle + (1/4!)\lambda^a\epsilon_{abcde}\Gamma^{bdce}|0\rangle$ where the vacuum $|0\rangle$ contains a spinorial index. The condition $\lambda^t C\Gamma^a\lambda = 0$ yields then that $\lambda_+\lambda^a \sim \epsilon^{abcde}\lambda_{bc}\lambda_{de}$, whereas $\lambda^t C\Gamma_a\lambda = 0$ yields $\lambda^a\lambda_{ab} = 0$. The solution of the former equation expresses λ^a in terms of λ^+ and λ_{ab}, and then the latter equation is also satisfied.

According to [1], physical states are defined as cohomology classes[3] of the nilpotent BRST-like operator

$$Q_B = \oint dz \lambda^\alpha d_{z\alpha} \tag{2.10}$$

where $d_{z\alpha}$ is defined in (2.2). Since $d_\alpha(z)d_\beta(w) \sim -(z-w)^{-1}\gamma^m_{\alpha\beta}\Pi_m(w)$, (2.9) implies that $\{Q_B, Q_B\} = 0$. The operator in (2.10) was used in [5] to show that the constraints of $D = (9,1), N = 1$ super-Yang-Mills theory or supergravity can be understood as integrability conditions on pure spinor lines.

Using the standard [14] free OPE's of the worldsheet fields[4]

$$p_\alpha(z)\theta^\beta(w) \sim \frac{\delta^\beta_\alpha}{z-w}\,, \qquad x^m(z)x^n(w) \sim -\eta^{mn}\log(z-w)\,, \tag{2.11}$$

one obtains the following BRST transformation rules

$$[Q_B, x^m] = \frac{1}{2}\lambda^\alpha\gamma^m_{\alpha\beta}\theta^\beta\,, \qquad \{Q_B, \theta^\alpha\} = \lambda^\alpha\,, \qquad [Q_B, \lambda^\alpha] = 0\,. \tag{2.12}$$

From these transformation rules the nilpotency of Q_B when (2.9) holds is obvious. For later use, we also need the variation of Π^m_z

$$[Q_B, \Pi^m] = [Q_B, (\partial x^m + \frac{1}{2}\theta^a\gamma^m_{\alpha\beta}\partial\theta^\beta)] = \lambda^\alpha\gamma^m_{\alpha\beta}\partial\theta^b\,. \tag{2.13}$$

Since the conjugate momenta p_α are independent variables, we must also determine their BRST variations

$$\{Q_B, p_\alpha\} = -\frac{1}{2}\partial x_m\gamma^m_{\alpha\beta}\lambda^\beta - \frac{3}{8}\gamma^m_{\alpha\beta}\partial\theta^\beta\lambda^\delta\gamma_{m\delta\gamma}\theta^\gamma - \frac{1}{8}\gamma^m_{\alpha\beta}\theta^\beta\partial\lambda^\delta\gamma_{m\delta\gamma}\theta^\gamma\,, \tag{2.14}$$

Since p_α is related to d_α (see eq. (2.2)), it is natural to use the latter as the fundamental variable, since its variation has a simpler form

$$\{Q_B, d_\alpha\} = -\Pi_m\gamma^m_{\alpha\beta}\lambda^\beta\,. \tag{2.15}$$

It is also natural to require compatibility of the supersymmetry transformations (2.5) with the symmetry generated by Q_B. Defining $[Q_\epsilon, \lambda^\alpha] = 0$, we have $[Q_\epsilon, Q_B] = 0$. This concludes our review of those results of Berkovits's program which we need.

[3]The cohomology of the operator Q should be computed in the infinite dimensional space of $x^m, \partial x^m, \partial\bar\partial x^m, \ldots, \theta^\alpha, \partial\theta^\alpha, \partial\bar\partial\theta^\alpha, \ldots, \lambda^\alpha, \partial\lambda^\alpha, \partial\bar\partial\lambda^\alpha, \ldots$, but one can restrict one's attention to particular subsets by using the usual filtration techniques (for example, the linear subspace of zero forms (functions) in (6.4)).

[4]As usual, we combine left- and right-moving parts of the fields of the open string into a single field on the double interval. This eliminates cross-terms such as $\ln(z-\bar w)$ from the propagators.

The BRST variation of x_m given in (2.12) is not nilpotent if λ does not satisfy any constraints. We therefore introduce a new anticommuting worldsheet-scalar spacetime-vector ξ^m, whose properties will be determined as we proceed, and define

$$[Q'_B, x^m] = \xi^m + \frac{1}{2}\lambda^\alpha \gamma^m_{\alpha\beta}\theta^\beta\,. \tag{2.16}$$

We find then $Q'_B = Q_B - \oint \xi^m \Pi_m dz$. Similarly, since (2.15) is not nilpotent, we introduce a new commuting worldsheet-scalar spacetime-spinor χ_α, and define

$$\{Q'_B, d_\alpha\} = \partial\chi_\alpha - \Pi_m \gamma^m_{\alpha\beta}\lambda^\beta + \xi_m \gamma^m_{\alpha\beta}\partial\theta^\beta\,. \tag{2.17}$$

This adds a term $-\oint \chi_\alpha \partial_z \theta^\alpha\, dz$ to Q'_B. The last term in (2.17) is induced by the extra term in (2.16). Since the BRST variation of the sum of the last two terms is a total derivative we introduced $\partial_z \chi_\alpha$ instead of a 1-form $\chi_{z\alpha}$. Requiring nilpotency of Q'_B on x_m and $d_{z\alpha}$ yields then the transformation laws for ξ^m and χ_α

$$\{Q'_B, \xi^m\} = -\frac{1}{2}\lambda^\alpha \gamma^m_{\alpha\beta}\lambda^\beta\,, \qquad [Q'_B, \chi_\alpha] = \xi_m \gamma^m_{\alpha\beta}\lambda^\beta\,. \tag{2.18}$$

The basic principle of our approach is to make the θ variation as simple as possible, hence we still define $\{Q'_B, \theta^\alpha\} = \lambda^\alpha$, and nilpotency on θ^α leads then to $[Q'_B, \lambda^\alpha] = 0$. Nilpotency on ξ^m is then obvious, but nilpotency on χ_α holds also as one may check by a Fierz transformation. The ghost numbers are assigned such that Q'_B has ghost number $+1$. Then x_m, θ^α and $d_{z\alpha}$ (or $p_{z\alpha}$) have ghost number zero, while ξ^m, λ^α and χ_α have ghost number $+1$. Finally, the transformation rule for Π^m becomes

$$[Q'_B, \Pi_m] = \partial\xi_m + \lambda^\alpha \gamma^m_{\alpha\beta}\partial\theta^\beta\,. \tag{2.19}$$

These modified BRST transformation rules can be incorporated in a modified BRST current. Introducing the antighosts β^m_z, κ^α_z, and $\beta_{z\alpha}$ for ξ_m, χ_α, and λ^α respectively, the BRST current takes the following form

$$j^{B\prime}_z \equiv \lambda^\alpha d_{z\alpha} - \xi^m \Pi_{zm} - \chi_\alpha \partial\theta^\alpha - \xi^m \kappa^\alpha_z \gamma_{m\alpha\beta}\lambda^\beta - \frac{1}{2}\lambda^\alpha \gamma^m_{\alpha\beta}\lambda^\beta \beta_{zm}\,. \tag{2.20}$$

Using the free OPE's, the OPE of two BRST currents can be evaluated

$$j^{B\prime}_z(z) j^{B\prime}_w(w) \sim \frac{1}{z-w}\Big(\xi^m \partial_w \xi_m + \lambda^\alpha \partial_w \chi_\alpha - \partial_w \lambda^\alpha \chi_\alpha\Big)\,. \tag{2.21}$$

The double poles cancel due to the statistics of the ghost fields. The nonvanishing of the OPE for $j^{B\prime}_z(z) j^{B\prime}_w(w)$ implies that the BRST charge is not nilpotent. Therefore, it does not make sense to study the cohomology of this Q. Another related problem is that the central charge of the stress tensor does not vanish. Both problems will be solved by a new ingredient.

The nonclosure of Q'_B is not very surprising. The BRST charge has the form

$$Q'_B = \oint dz\,\Big(C^M(z) W_{zM}(z) - \frac{1}{2} f_{MN}{}^R B_{zR}(z) C^N(z) C^M(z)\Big)\,, \tag{2.22}$$

where $W_{zM}(z)$ are local generators of a (super) Kac-Moody algebra, f_{MNR} are the structure constants of the underlying (super) Lie algebra, and $C^M(z)$ and $B_z^M(z)$ are the ghosts and the antighosts, respectively, associated to the generators $W_{zM}(z)$. However, our algebra [10] has a central charge: identifying $W_{zM} = \{\Pi_z^m, d_{z\alpha}, \partial_z\theta^\alpha\}$, we have

$$W_{zM}(z)W_{wN}(w) \sim \frac{f_{MN}{}^R}{z-w} W_{wR}(w) + \frac{h_{MN}}{(z-w)^2} \tag{2.23}$$

where

$$h_{MN} = \begin{pmatrix} -\eta^{mn} & 0 & 0 \\ 0 & 0 & \delta_\alpha^\beta \\ 0 & -\delta_\beta^\alpha & 0 \end{pmatrix} \tag{2.24}$$

and the non-trivial $f_{MN}{}^R$ are only $f_{\alpha\beta}{}^m = -\gamma^m_{\alpha\beta}$, and $f_{\alpha m\beta} = -f_{m\alpha\beta} = \gamma_{m\alpha\beta}$. The structure constants $f_{MN}{}^R$ and the invariant metric h_{MN} satisfy the usual algebraic identities

$$f_{MN}{}^R f_{RP}{}^Q + \text{cyclic perm's} = 0\,, \qquad f_{MP}{}^R h_{RN} \pm f_{NP}{}^R h_{MR} = 0\,. \tag{2.25}$$

The Jacobi identities follow from the Fierz identities for the Dirac matrices.

To construct a nilpotent BRST charge, we promote the central charge to an operator and we introduce a new anticommuting $b - c_z$ system associated to this operator. The field b is a 0-form with ghost number -1 and conformal spin 0, and c_z is a 1-form with ghost number 1 and conformal spin 1. They have the OPE

$$c_z(z)b(w) \sim \frac{1}{z-w}\,. \tag{2.26}$$

Following [13], but with the full field b instead of only the zero mode b_0, the correct BRST current $j_z^B(z)$ is then given by[5]

$$\begin{aligned} j_z^B =\ & \lambda^\alpha d_{z\alpha} - \xi^m \Pi_{zm} - \chi_\alpha \partial\theta^\alpha - \xi^m \kappa_z^\alpha \gamma_{m\alpha\beta}\lambda^\beta - \frac{1}{2}\lambda^\alpha\gamma^m_{\alpha\beta}\lambda^\beta\beta_{zm} + \\ & + c_z - \frac{1}{2} b \left(\xi^m \partial_z \xi_m - \frac{3}{2}\chi_\alpha\partial_z\lambda^\alpha + \frac{1}{2}\partial_z\chi_\alpha\lambda^\alpha \right) - \frac{1}{2}\partial_z\left(b\,\chi_\alpha\lambda^\alpha\right). \end{aligned} \tag{2.27}$$

The last term is added to make this current nilpotent

$$j_z^B(z) j_w^B(w) \sim 0\,. \tag{2.28}$$

It is easy to check that the contributions to (2.28) from the central charge in (2.23) are cancelled by the contributions to (2.28) due to (2.26).

Because the new BRST charge $Q = \oint dz j_z^B$ is nilpotent its cohomology is well-defined. We postpone the derivation of the transformation rules of the antighosts until we discuss the BRST invariance of the quantum action.

[5] Another nilpotent charge depending also on b and c_z is given by

$$\begin{aligned} Q = \oint dz \Big[& \lambda^\alpha d_{z\alpha} - \xi^m \Pi_{zm} - \chi_\alpha \partial\theta^\alpha - \xi^m \kappa_z^\alpha \gamma_{m\alpha\beta}\lambda^\beta - \frac{1}{2}\lambda^\alpha\gamma^m_{\alpha\beta}\lambda^\beta\beta_{zm} + c_z - \\ & - \frac{1}{2} b (\xi^m \partial_z\xi_m - \chi_\alpha\partial_z\lambda^\alpha + \partial_z\chi_\alpha\lambda^\alpha) + \frac{1}{4} b\,\partial b\,\xi^m \lambda^\alpha \gamma_{m\alpha\beta}\lambda^\beta \Big]\,. \end{aligned}$$

3. Action and boundary conditions

The dynamics of the model is fixed by postulating free OPE's for all fields. It is natural to ask whether the free action is BRST invariant. This represents a consistency check between the structure of the BRST charge and the free OPE's used to construct it. For the closed string we impose as usual periodic boundary conditions. For the open string we shall in this way derive the boundary conditions on the fields. The proof that the action is invariant is equivalent to the statement that the two-point Ward identities are satisfied at the tree graph level.

Consider the tree level action

$$S = \int d^2z \Big(\frac{1}{2}\partial x^m \bar{\partial} x_m + p_\alpha \bar{\partial}\theta^\alpha + \hat{p}_\alpha \partial \hat{\theta}^\alpha\Big)\,. \tag{3.1}$$

We perform the BRST variation[6] of the left-movers

$$\begin{aligned}
[Q, S_L] = \int d^2z \Bigg[& \left(\partial\xi^m + \frac{1}{2}\partial\lambda\gamma^m\theta + \frac{1}{2}\lambda\gamma^m\partial\theta\right)\bar{\partial}x^m + \\
& + \Bigg(\partial\chi_\alpha - \frac{1}{2}\partial x^m(\gamma_m\lambda)_\alpha + \frac{1}{2}\partial\xi^m(\gamma_m\theta)_\alpha + \xi^m(\gamma_m\partial\theta)_\alpha - \\
& - \frac{1}{8}(\gamma^m\theta)_\alpha(\theta\gamma_m\partial\lambda) - \frac{3}{8}(\gamma^m\partial\theta)_\alpha(\theta\gamma_m\lambda)\Bigg)\bar{\partial}\theta^\alpha - p_\alpha\bar{\partial}\lambda^\alpha\Bigg] \\
= \int d^2z \Bigg[& \left(\partial x^m + \frac{1}{2}\theta\gamma^m\partial\theta\right)\bar{\partial}\xi_m + \partial\theta^\alpha\bar{\partial}\chi_\alpha - \\
& - \left(p_\alpha - \frac{1}{2}\partial x^m(\gamma_m\theta)_\alpha - \frac{1}{8}(\gamma^m\theta)_\alpha(\theta\gamma_m\partial\theta)\right)\bar{\partial}\lambda^\alpha\Bigg]\,.
\end{aligned} \tag{3.2}$$

To obtain this result we partially integrated the BRST variation of the action to bring the $\bar{\partial}$ derivatives on ξ^m, λ^α and χ_α, because then the variations can be canceled by suitable BRST variations of β_m, β_α and κ^α.

The boundary terms which we produce in this way are

$$\begin{aligned}
& \frac{1}{2}\partial\left(\xi^m\bar{\partial}x_m\right) - \frac{1}{2}\bar{\partial}\left(\xi^m\partial x_m\right) + \frac{1}{4}\partial\left((\lambda\gamma^m\theta)\bar{\partial}x_m\right) - \\
& - \frac{1}{4}\bar{\partial}\left((\lambda\gamma^m\theta)\partial x_m\right) + \frac{1}{2}\partial\left(\xi^m(\theta\gamma_m\bar{\partial}\theta)\right) - \frac{1}{2}\bar{\partial}\left(\xi^m(\theta\gamma_m\partial\theta)\right) - \\
& - \frac{1}{8}\partial\left((\bar{\partial}\theta\gamma^m\theta)(\theta\gamma_m\lambda)\right) + \frac{1}{8}\bar{\partial}\left((\partial\theta\gamma^m\theta)(\theta\gamma_m\lambda)\right)\,.
\end{aligned} \tag{3.3}$$

To these terms we should add the corresponding terms from the antiholomorphic sector.

[6] In the rest of the paper we often omit spinor indices; when needed we use parentheses as in $(A\gamma^m B)$ to denote contraction of spinors indices.

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Next, we replace the total derivatives ∂ and $\bar{\partial}$ by $(1/2)\partial_\sigma$ and find then the following conditions at $\sigma = 0, \pi$ for the BRST symmetry of the open string

$$\begin{aligned}
\xi^m \partial_\tau x_m &= \hat{\xi}^m \partial_\tau x_m \,, \\
\chi_\alpha \partial_\tau \theta^\alpha &= \hat{\chi}_\alpha \partial_\tau \hat{\theta}^\alpha \,, \\
\lambda \gamma^m \theta &= \hat{\lambda} \gamma^m \hat{\theta} \,, \\
(\partial_\tau \theta \gamma^m \theta)(\theta \gamma_m \lambda) &= (\partial_\tau \hat{\theta} \gamma^m \hat{\theta})(\hat{\theta} \gamma_m \hat{\lambda}) \,.
\end{aligned} \tag{3.4}$$

A solution of all these boundary conditions, assuming that $\theta^\alpha = \hat{\theta}^a$, is

$$\theta^\alpha = \hat{\theta}^\alpha \,, \qquad \xi^m = \hat{\xi}^m \,, \qquad \chi_\alpha = \hat{\chi}_\alpha \,, \qquad \lambda^\alpha = \hat{\lambda}^\alpha \,, \qquad \epsilon^\alpha = \hat{\epsilon}^\alpha \,. \tag{3.5}$$

The BRST variation of the classical action in (3.2) can be written in a simpler form if we use the conjugate momenta Π^m and d_α,

$$[Q, S_L] = \int d^2 z \left(\Pi^m \bar{\partial} \xi_m + \partial \theta^\alpha \bar{\partial} \chi_\alpha - d_\alpha \bar{\partial} \lambda^\alpha \right) \,. \tag{3.6}$$

Since it is proportional to the equations of motions of the ghost fields, it can be compensated by adding a ghost-antighost action

$$S_{L,\mathrm{ghost}} = \int d^2 z \left(\beta_{zm} \bar{\partial} \xi^m + \beta_{z\alpha} \bar{\partial} \lambda^\alpha + \kappa_z^\alpha \bar{\partial} \chi_\alpha + c_z \, \bar{\partial} b \right) \,, \tag{3.7}$$

and defining the variations of the antighost fields $\beta_{zm}, \beta_{z\alpha}, \kappa_z^\alpha$ and b in a suitable way. These transformation rules contain various terms bilinear in the ghosts and antighosts, needed to cancel the variations of the ghosts in (3.7). An easy way to obtain the transformation rules of the antighosts is to use the BRST charge as we now discuss.

From the BRST charge we derive the following transformation rules for the antighost fields

$$\begin{aligned}
\{Q, b\} &= 1 \,, \\
\{Q, \beta_z^m\} &= -\Pi_z^m - \kappa_z \gamma^m \lambda + b \, \partial_z \xi^m + \frac{1}{2} (\partial_z b) \, \xi^m \,, \\
[Q, \kappa_z^\alpha] &= -\partial_z \theta^\alpha + b \, \partial_z \lambda^\alpha + \frac{1}{4} (\partial_z b) \, \lambda^\alpha \,, \\
[Q, \beta_{z\alpha}] &= d_{z\alpha} - \beta_z^m (\gamma_m \lambda)_\alpha - \xi^m (\gamma_m \kappa_z)_\alpha - b \, \partial_z \chi_\alpha - \frac{3}{4} (\partial_z b) \, \chi_\alpha \,.
\end{aligned} \tag{3.8}$$

For the new ghost c_z, we find

$$\{Q, c_z\} = -\frac{1}{2} \left(\xi^m \partial_z \xi_m - \frac{3}{2} \chi_\alpha \partial_z \lambda^\alpha + \frac{1}{2} \partial_z \chi_\alpha \lambda^\alpha \right) \,. \tag{3.9}$$

Using again integration by parts and Fierz identities, it is easy to show that the action is BRST invariant

$$[Q, S_L + S_R + S_{L,\mathrm{ghost}} + S_{R,\mathrm{ghost}}] = 0 \,.$$

In the case of the open string, taking (3.5) into account, the last manipulations lead to the following boundary terms

$$\partial_{\underline{\mathbf{s}}}\left[(b-\hat{b})\left(\xi^m\partial_\tau\xi^m-\frac{3}{2}\chi_\alpha\partial_\tau\lambda^\alpha+\frac{1}{2}\lambda^\alpha\partial_\tau\chi_\alpha\right)\right] \tag{3.10}$$

whose solution is given by $b=\hat{b}$ at $\sigma=0,\pi$.

To derive the equations of motion of the antighost fields $\beta_z^\mu, \beta_\alpha$ and κ_z^α, one has to integrate the action by parts. This implies new boundary conditions on the antighost fields [15] which take the form

$$\beta_{\underline{\mathbf{s}}}^m=\hat{\beta}_{\underline{\mathbf{s}}}^m\,,\qquad \beta_{\underline{\mathbf{s}}\alpha}=\hat{\beta}_{\underline{\mathbf{s}}\alpha}\,,\qquad \kappa_{\underline{\mathbf{s}}}^\alpha=\hat{\kappa}_{\underline{\mathbf{s}}}^\alpha\,, c_{\underline{\mathbf{s}}}=\hat{c}_{\underline{\mathbf{s}}}\,,\qquad \text{at}\qquad \underline{\mathbf{s}}=0,\pi\,. \tag{3.11}$$

Notice that since all ghost fields are supersymmetrically invariant, the new terms in the action do not spoil the invariance of the theory under supersymmetry transformations. However, for the open string we must make sure that the sum of the boundary terms, which are produced by susy variations, cancels [15]. For susy we find the following boundary terms

$$\begin{aligned}\delta_\epsilon\mathcal{L}=&\frac{1}{2}\partial\left[-\frac{1}{2}(\epsilon\gamma^m\theta)\bar{\partial}x_m\right]+\frac{1}{2}\bar{\partial}\left[\frac{1}{2}(\epsilon\gamma^m\theta)\partial x_m\right]+\\&+\frac{1}{2}\partial\left[-\frac{1}{2}(\hat{\epsilon}\gamma^m\hat{\theta})\bar{\partial}x_m\right]+\frac{1}{2}\bar{\partial}\left[\frac{1}{2}(\hat{\epsilon}\gamma^m\hat{\theta})\partial x_m\right]\,,\end{aligned} \tag{3.12}$$

and further terms with three θ's. Replacing the total derivative ∂ and $\bar{\partial}$ by $(1/2)\partial_\sigma$ one finds the combination

$$\partial_{\underline{\mathbf{s}}}\left[(\epsilon\gamma^m\theta)\partial_\tau x_m-(\hat{\epsilon}\gamma^m\hat{\theta})\partial_\tau x_m\right]. \tag{3.13}$$

Since $\partial_\tau x_m$ does not satisfy any conditions, we find as condition for susy of the open string

$$(\epsilon\gamma^m\theta)=(\hat{\epsilon}\gamma^m\hat{\theta})\qquad \text{at}\qquad \sigma=0,\pi\,. \tag{3.14}$$

Similarly, the terms with three θ's yield

$$(\partial\theta\gamma^m\bar{\partial}\theta)(\epsilon\gamma_m\theta)=(\partial\hat{\theta}\gamma^m\bar{\partial}\hat{\theta})(\hat{\epsilon}\gamma_m\hat{\theta})\,. \tag{3.15}$$

The solution of these conditions is $\epsilon=\hat{\epsilon}$.

4. The energy-momentum tensor, the field B_{zz} and the ghost current

The antighost b_{zz} for world-sheet diffeomorphisms of the bosonic string and the RNS version of the superstring plays an important role in the construction of higher-loop amplitudes and in the parametrization of moduli of Riemann surfaces. We believe, therefore, that also for the superstring a composite field B_{zz}, constructed from the fundamental fields of the world-sheet theory and playing the same role as the fundamental b-field, must exist. In addition, in order to maintain an explicit covariant formulation, this composite field B_{zz} should be invariant under super-Poincaré transformations of the target space.

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Since the theory is a free conformal field theory, it is easy to write down the proper energy-momentum tensor. It follows from the action and is given by the formula

$$T_{zz} = -\frac{1}{2}\Pi^m_z \Pi_{mz} - d_{z\alpha}\partial_z\theta^\alpha - \beta_{zm}\partial_z\xi^m - \beta_{z\alpha}\partial_z\lambda^\alpha - \kappa^\alpha_z\partial_z\chi_\alpha + \partial_z b\, c_z\,. \qquad (4.1)$$

This current T_{zz} is BRST invariant, as can be directly verified by using the BRST transformation rules of the fields (there are no double contractions between Q and pairs of fields in (4.1))

$$j^B_z(z) T_{ww}(w) \sim \frac{j^B_w(w)}{(z-w)^2}\,. \qquad (4.2)$$

The fact that T_{zz} is BRST invariant is encouraging because the composite field B_{zz} should satisfy the relation $T_{zz} = \{Q, B_{zz}\}$. The first two terms in T_{zz} can be rewritten as $-(1/2)\partial_z x^m \partial_z x^m - p_{z\alpha}\partial_z\theta^\alpha$, as follows from the definition of Π^m and $d_{z\alpha}$. In the case of open strings, T_{zz} should share boundary conditions with the antiholomorphic partner $\bar{T}_{\bar{z}\bar{z}}$. Due to the boundary conditions (3.5), (3.10) and (3.11), T and $\bar{T}$ can be combined into a single expression which is defined in the whole complex z-plane.

One can compute the central charge in the OPE of T_{zz} with itself, using the free OPE's

$$T_{zz}(z) T_{ww}(w) \sim \frac{20}{(z-w)^4} + \frac{2}{(z-w)^2} T_{ww}(w) + \frac{1}{(z-w)}\partial_w T_{ww}(w) + \cdots\,. \qquad (4.3)$$

The value of the central charge is obtained by summing all the contributions: $(+10\times 1)_x + (-16\times 2)_{p,\theta} + (-10\times 2)_{\xi,\beta} + (+16\times 2)_{\lambda,\beta} + (+16\times 2)_{\chi,\kappa} + (-1\times 2)_{b,c} = 20$. Here $(n\times m)_{A,B}$ denotes the contribution from an A,B system, with n the number of components of the fields, and m the central charge for a single field. Relative signs are given by the statistics of the fields.

The resulting central charge does not vanish and, therefore, the conformal symmetry is lost. However, because $20 = 10\times 2$, we can compensate the non-vanishing central charge by an additional anticommuting spacetime-vector spin 0 -spin 1 system ω^m_z, η^m. The final energy momentum tensor is then given by

$$T_{zz} \to T_{zz} + \partial_z\eta_m\, \omega^m_z\,. \qquad (4.4)$$

Due to the contribution from the ω^m_z, η^m system the central charge now vanishes. In order not to spoil the rest of the construction, we assume that these new fields are inert under the BRST symmetry and supersymmetry. In the following, we will argue that these new fields are needed for maintaining complete covariance, and in the next section we show how they appear when one relaxes the pure spinor constraint.

Another important operator in the construction of the superstring amplitudes is the ghost current. In the present formalism, we have the following expression

$$J^{gh}_z = -\left(\beta_{mz}\xi^m + \kappa^\alpha_z\chi_\alpha + \beta_{z\alpha}\lambda^\alpha + b\, c_z + \eta^m\omega_{zm}\right), \qquad (4.5)$$

which satisfies

$$j^B_z(z) J^{gh}_w(w) \sim \frac{-j^B_z(w)}{(z-w)}\,. \qquad (4.6)$$

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Using the free OPE's one can compute the anomaly in the ghost current J_z and obtains

$$J^{gh}_z(z) J^{gh}_w(w) \sim \frac{c/3}{(z-w)^2} = \frac{-11}{(z-w)^2}\,. \tag{4.7}$$

The anomaly is obtained summing all the contributions: $(+10)_{\xi,\beta} + (-16)_{\lambda,\beta} + (-16)_{\chi,\kappa} + (+1)_{b,c} + (+10)_{\eta\omega} = -11$ and the OPE's with $T_{zz}(z)$ is given by

$$T_{zz}(z) J^{gh}_w(w) \sim \frac{11}{(z-w)^3} + \frac{J^{gh}_w(w)}{(z-w)^2} + \frac{\partial_w J^{gh}_w}{(z-w)}\,. \tag{4.8}$$

Following the idea that the theory should be described by a topological field theory in two dimensions, see the appendix, we only need the BRST charge, the energy-momentum tensor, a U(1) charge and, finally, an antighost field B_{zz} to construct amplitudes (for a pedagogical explanation see [16]). The last one is a spin-2 worldsheet-tensor (spacetime-scalar) with ghost number -1. It should satisfy the condition that it has no singular OPE with itself [17] and $T_{zz} = \{Q, B_{zz}\}$. In addition, to obtain a complete twisted $N=2$ superconformal algebra, the OPE with the BRST current should be given by

$$j^B_z(z) B_{ww}(w) \sim \frac{2\,c/3}{(z-w)^3} + \frac{2\,J^{gh}(w)}{(z-w)^2} + \frac{2\,T(w)}{z-w}\,, \tag{4.9}$$

where T is given in (4.1), (4.3), and (4.4) the ghost current $J^{gh}(w)$ is given in (4.5), and $2\,c/3 = -22$.

Notice that in our framework B_{zz} is a composite field, in contrast to the RNS formulation. The requirement that B_{zz} has no singular terms with itself, and the requirement that $T_{zz} = \{Q, B_{zz}\}$, put severe restrictions on the possible solutions for B_{zz}.

Since the b field is a Grassmann variable, we can decompose the composite operator B_{zz} into

$$B_{zz} = B^{(-1)}_{zz} + b\,B^{(0)}_{zz} + \partial_z b\,B^{(0)}_z + \partial^2_z b B^{(0)} + b\,\partial_z b\,B^{(1)}_z\,, \tag{4.10}$$

where the upper index denotes the ghost charge of B_{zz}. The condition $T_{zz} = \{Q, B_{zz}\}$ leads to a solution

$$\begin{aligned} B^{(-1)}_{zz} &= \left(\beta^m_z \Pi_{mz} - \beta_{z\alpha}\partial_z\theta^\alpha - \kappa^\alpha_z d_{\alpha z}\right)\,, \\ B^{(0)}_{zz} &= \frac{1}{2}\Pi^m_z \Pi_{mz} + d_{z\alpha}\partial_z\theta^\alpha - \partial_z\eta_m\,\omega^m_z\,, \\ B^{(0)}_z &= \frac{1}{2}\left(-\beta^m_z\xi_m - \frac{1}{2}\beta_{z\alpha}\lambda^\alpha - \frac{3}{2}\kappa^\alpha_z\chi_\alpha\right)\,, \\ B^{(0)} &= 0\,, \\ B^{(1)}_z &= c_z\,. \end{aligned} \tag{4.11}$$

This proposal for B_{zz} is, however, not nilpotent.

The requirement that B_{zz} satisfies $T_{zz} = \{Q, B_{zz}\}$ is not enough to fix B_{zz} completely. In fact, one can always add a trivial part $B_{zz} \to B_{zz} + [Q, X_{zz}]$ where X_{zz} is a local polynomial with ghost number -2. Using this freedom, we construct other expressions that enjoy different algebraic properties. Finally, we show that one particular realization of B_{zz} corresponds to the usual BRST charge for the bosonic string (there are no world-sheet fermions in our theory).

For instance, another expression for B_{zz} is given by

$$B_{zz} = B_{zz}^{(-1)} + b\, B_{zz}^{(0)} + b\,\partial_z b\, B_z^{(1)}\,, \tag{4.12}$$

where now

$$\begin{aligned} B_{zz}^{(-1)} &= \frac{1}{2}\left(\beta_z^m \Pi_{mz} - \beta_{z\alpha}\partial_z\theta^\alpha - \kappa_z^\alpha d_{\alpha z}\right)\,, \\ B_{zz}^{(0)} &= -\frac{1}{2}\Big(\beta_z^m \partial_z \xi_m + \beta_\alpha \partial_z \lambda^a + \kappa^\alpha \partial_z \chi_a - 2\,\partial_z \eta_m\, \omega_z^m\Big)\,, \\ B_z^{(1)} &= -\frac{1}{4}\left(\Pi_z^m \xi_m - \frac{1}{2} d_{z\alpha}\lambda^\alpha + \frac{3}{2}\partial_z\theta_z^\alpha \chi_\alpha\right) + c_z\,. \end{aligned} \tag{4.13}$$

This expression again satisfies $T_{zz} = \{Q, B_{zz}\}$. It is easy to check that the OPE with the BRST current $j_z^B(z)$ produces double poles which are field dependent. The bilinear part of these field-dependent double poles coincides with the ghost current (4.5). However, the other singularities in this OPE shows that the B_{zz} in (4.12) and in (4.13) has neither the OPE in (4.9) with $j_z^B(z)$ nor is nilpotent.

A third proposal for B_{zz} has much better properties than the previous two proposals. It differs again from the previous ones by trivial BRST terms and is given by

$$B_{zz}(z) = bT_{zz}(z) + \partial_z b\,\left(J_z^{gh}(z) + b\, j_z^B(z)\right) + \alpha \partial_z^2 b\,. \tag{4.14}$$

It satisfies the OPE in (4.9) with $c = 0$. Moreover it is a primary field for $\alpha = 35/2$ However, it is still not nilpotent since it has a singular OPE with itself. Now observe that the terms proportional to $\partial_z b$ can be written as the BRST variation of $b\, J_z(z)$ using (4.6). Furthermore, $\partial_z b$ is BRST invariant. Therefore, we can eliminate these terms from $B_{zz}(z)$ to arrive to a remarkably simple expression. Consider the following proposal for B_{zz}

$$B_{zz}(z) = bT_{zz}(z) + \alpha \partial_z^2 b\,. \tag{4.15}$$

This new composite operator satisfies $T_{zz}(z) = \{Q, B_{zz}(z)\}$, since $\{Q, b\} = 1$. Computing the OPE's of B_{zz} with the ghost charge and with the energy-momentum tensor, one can check that it is a ghost number -1, anticommuting primary field with spin 2 if $\alpha = -(1/2)$. By extracting the term proportional to $\partial_z b c_z$ in T_{zz}, we end up with the expression

$$B_{zz}(z) = b\,\hat{T}_{zz}(z) + b\partial_z b c_z - \frac{1}{2}\partial_z^2 b\,, \tag{4.16}$$

where $\hat{T}_{zz}(z)$ is given by (4.1) without the last term. The new field $B_{zz}(z)$ satisfies

$$B_{zz}(z) B_{ww}(w) \sim \frac{2}{z-w}(\partial_w^2 b\, \partial_w b)(w)\,, \tag{4.17}$$

where the r.h.s. can be interpreted as an anomaly (just like the nonclosure term $\partial c^z \partial^2 c^z / (z-w)$ in the product of two BRST currents in the RNS approach is an anomaly). The operator $\oint B_{zz}(z) dz$ has the same structure (with $c \to b$) as the BRST charge of a bosonic string whose matter part is represented not only by the complete set of superstring matter fields $x^m, d_{z\alpha}$ and θ^α , but also by the ghost fields ξ^m, λ^α and χ_α, ω_z^m. To complete the correspondence with the usual bosonic BRST charge, one has to view the fields b, c_z as the diffeomorphism ghosts c^z, b_{zz} of the usual formulation. This follows from twisting, as we now explain.

The b, c_z system which we introduced to take care of the central charge in the Kac-Moody algebra is related to the diffeomorphism ghosts b_{zz}, c^z of the RNS by twisting twice: once by $T_{bc} = \partial_z b c_z \to T' = T - (1/2)\partial_z j_z$ where $j_z = -bc_z$ which gives $c_z \to \psi_{3/2}$ and $b \to \psi_{-(1/2)}$, and once more by $T'' \to T' - (1/2)\partial_z j'_z$ with $j' = -\psi_{-1/2}\psi_{(3/2)}$ which gives $\psi_{-1/2} \to c^z$ and $\psi_{3/2} \to b_{zz}$. Applying the twisting $b \to c^z$ and $c_z \to b_{zz}$ to the field B_{zz} in (4.16), one obtains the usual BRST charge of the bosonic string, except that the central charge of $\hat{T}_{zz}$ is $+2$ instead of $+26$ and the coefficient of $\partial^2 c^z$ is $-1/2$ instead of $-3/2$.

Conversely, our BRST charge is unitarily equivalent to $\oint dz\, c_z$ which is mapped, by twisting, into $\oint dz\, b_{zz}$. Hence, our $\oint B_{zz} dz$ and Q are mapped under twisting into Q_{RNS} and $\oint dz\, b_{zz}$, respectively. On the other hand, in our approach, the charge $\oint B_{zz} dz$ should contribute to the measure for higher-loop calculations [18] in a similar way as $\oint dz\, b_{zz}$ is used in the RNS approach.

As is well known, in the RNS formulation (or in the bosonic string), the vertex operators for the open string are defined as the cohomological classes of the BRST operator. They can be separated into unintegrated $\mathcal{U}^{(1)}$ (with ghost number $+1$) and integrated ones $\oint \mathcal{V}_z^{(0)} dz$ (with ghost number 0) which satisfy the equations

$$[Q, \mathcal{U}^{(1)}] = 0\,, \qquad \Big[Q, \mathcal{V}_z^{(0)}\Big] = \partial_z \mathcal{U}^{(1)}\,. \tag{4.18}$$

Defining a new operator δ_z such that $\partial_z = [Q, \delta_z]$, one has: $[Q, \mathcal{V}_z^{(0)}] = \Big[[Q, \delta_z], \mathcal{U}^{(1)}\Big] = \Big[Q, [\delta_z, \mathcal{U}^{(1)}]\Big]$, where the first equation has been used. This leads to

$$\Big[Q, \mathcal{V}_z^{(0)} - [\delta_z, \mathcal{U}^{(1)}]\Big] = 0\,, \tag{4.19}$$

whose solution is $\mathcal{V}_z^{(0)} = [\delta_z, \mathcal{U}^{(1)}] + [Q, \mathcal{N}_z^{(-1)}]$. Indeed, there is no nontrivial cohomology in this sector because Q can be mapped into $\oint dz\, c_z$ by a similarity transformation, see the section on comments, and the solution of $\oint dz\, c_z |\psi\rangle = 0$ is $|\psi\rangle = \oint c_z |\phi\rangle$. This means that given an unintegrated vertex $\mathcal{U}^{(1)}$, one is able to construct the integrated one, namely $\mathcal{V}_z^{(0)}$, by acting with δ_z, up to a BRST trivial part, denoted here by $\mathcal{N}_z^{(-1)}$. By simple manipulations, it is easy to verify that $\delta_z = \oint dz\, B_{zz}$. Hence

$$\mathcal{V}_z^{(0)} = \oint dz B_{zz}\, \mathcal{U}^{(1)} + \{Q, N_z^{(-1)}\}\,, \tag{4.20}$$

and we can directly check that $[Q, \oint dz \mathcal{V}_z^{(0)}] = 0$.[7] This confirms that the field B_{zz} constructed out the fundamental fields of the superstring plays the same role as the RNS b-fields, and therefore it can be used to fix the moduli of the Riemann surfaces.

[7]Recall that in the RNS approach $[Q, (z\, b_{-1} - b_0)\mathcal{V}_z(z)] = [Q, \oint dw((z-w)\, b(w))\mathcal{V}_z(z)]$ and act with Q on $b(w)$ and $\mathcal{V}_z(z)$. Double contractions from Q to $b(w)$ and $\mathcal{V}_z(z)$ vanish. One obtains $-\mathcal{V}_z(z) + b_{-1}\mathcal{U} + \partial(b_0\mathcal{U} - z b_{-1}\mathcal{U})$. This yields $\mathcal{V}_z(z) = b_{-1}\mathcal{U} - [Q, (z\, b_{-1} - b_0)\mathcal{V}_z(z)] - \partial[(z\, b_{-1} - b_0)\mathcal{U}(z)]$ which agrees with our (4.20).

5. Relation with the RNS superstring

In the previous section we suggested an indentification of the ghost b, c_z with the ghosts c^z, b_{zz} of the RNS approach. In the present section, we will provide further relations between the RNS superstring and the present formulation. Some aspects of this problem have been already discussed in [18] and [21]. In the following, we decompose SO(9, 1) spinors and vectors with respect to the U(5)-like subgroup as follows

$$\theta^\alpha = \left(\theta_+, \theta^a, \theta_{[ab]}\right), \qquad p_\alpha = \left(p^+, p_a, p^{[ab]}\right), \qquad x^m = (x^a, x_a), \tag{5.1}$$

where θ_+ and p^+ are opposite-chirality U(5) singlets, θ^a and x^a are vectors in the $\underline{5}$, and p_a and x_a are vectors in the $\underline{5}^*$. The components $\theta_{[ab]}$ and $p^{[ab]}$ are tensors in the $\underline{10}$ and $\underline{10}^*$, respectively.

Step 0: we start from the RNS fields $(x^m, \psi^m, b, c, \beta, \gamma)$, where (b, c) are the conventional diffeomorphism ghosts and (β, γ) are their superpartners. In total, we have 12 bosonic variables and 12 fermionic ones. As shown in [21], these fields can be mapped into a subset of the Green-Schwarz variables: $(x^m, \theta_+, p^+, \theta^a, p_a, \beta^+, \lambda_+)$. The opposite chirality fields p^+, p_a, describe the six momenta conjugate to θ_+, θ^α. The two chiral bosons β^+, λ_+ take into account the contribution of the (β, γ) system, while p^+, θ_+ take into account (b_{zz}, c^z). The worldsheet RNS fermions ψ^m are decomposed into ψ_a, ψ^a and are mapped into the spacetime fermions p_a, θ^a. We should point out that this step requires intermediate bosonization and fermionization of (b, c) and of (β, γ) (see also [16] for the D=6 case and [19, 20] for Calabi-Yau and $K3$ compactifications), with $b^+ = e^t$ and $\lambda_+ = e^s$.

Step 1: in order to form a complete Green-Schwarz fermion θ^α, we have to add the tensor components $\theta_{[ab]}$ and their conjugated momenta $p^{[ab]}$. This can be done in a "topological" way, by adding a *BRST quartet* consisting of the tensors $\theta_{[ab]}$ and $p^{[ab]}$, and containing a further set of tensor fields $(v^{[ab]}, u_{[ab]})$ with opposite statistic. By topological we mean that the BRST charge associated to them is given by

$$Q = \oint dz\, u_{[ab]} p^{[ab]}, \tag{5.2}$$

which implies that, postulating free OPE's for all the systems, $\{Q, \theta_{[ab]}\} = u_{[ab]}$ and $[Q, v^{[ab]}] = p^{[ab]}$. Then nilpotency requires $[Q, u_{[ab]}] = 0$ and $\{Q, p^{[ab]}\} = 0$.

Together with the chiral boson s, the spinor $\lambda^\alpha = (e^s, u^a, u_{[ab]})$ forms a pure spinor (see (2.9)) if $u^a = -(1\ 8)e^{-s}\epsilon^{abcde}u_{[bc]}u_{[de]}$ [18]. The conjugate momenta $\beta_\alpha = (e^t, 0, v^{[ab]})$ are identified with the eleven independent components of the pure spinor λ^α. Counting the degrees of freedom, the number of bosons minus fermions cancels again as in RNS superstring. The present setting coincides with the pure spinor formulation [1], the BRST charge is nilpotent and the Lorentz algebra can be computed in terms of covariant combinations of pure spinors.

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Step 2: in order to arrive at a completely covariant formulation, we let all the components of the spinor λ^α become independent. Namely we want to remove the pure spinor constraint by introducing a new independent field u^a and its conjugate momentum β_a in order to reconstruct a complete spinor. Again, we introduce a BRST trivial quartet consisting of the spinor parts u^a and v_a, and half of a spacetime vector ξ^a and its conjugate momentum β_a. Notice that the statistics of the new fields should be opposite to those of β_α and λ^α, and, therefore, ξ^a and β_a are anticommuting vectors.

We can also understand the necessity of the new degrees of freedom by observing that, relaxing the pure spinor constraint $\lambda\gamma^a\lambda = 0$ is equivalent to set

$$\{Q, \xi^m\} = -\frac{1}{2}\lambda\gamma^m\lambda\,. \tag{5.3}$$

(Recall that $\lambda\gamma_a\lambda = 0$ is automatically satisfied if $\lambda\gamma^a\lambda = 0$ is satisfied). The $\underline{5}$ part of this vectorial equation can be solved in terms of u^a:

$$u^a = e^{-s}\left(\{Q, \xi^a\} - \frac{1}{8}\epsilon^{abcde}u_{[bc]}u_{[de]}\right). \tag{5.4}$$

Then, finally substituting u^a into the $\underline{5}^*$ part of equation (5.3), one gets $\{Q, \xi_a\} = e^{-s}\{Q, \xi^b\}u_{[ba]}$ which implies that the vector ξ^m is a constrained vector. The new fields u^a, v_a and ξ^a, β_a form again a BRST quartet.

Step 3: to remove the vectorial constraint, we can further enlarge the field space by introducing another quartet formed by the missing components of the vector (ξ_a, β^a) (removing the constraint) and introducing a new vector χ_a with its conjugate momentum κ^a. The latter can be viewed as the $\underline{5}$-part of χ_α and of its conjugate momentum κ^α. As above, the introduction of these new fields does not affect the physics of the theory.

Step 4: all the constrains are now removed, however the spinor χ_α and its conjugate momentum are treated non-covariantly. A completely covariant approach requires 11 more anticommuting fields. One of these new fields, namely the scalar $b - c_z$ system, already entered the formalism by requiring the nilpotency of the BRST charge following [13]. The further 10 fields ω_z^m and η_m are necessary for the vanishing of the central charge. Together with the $\underline{1}$- and $\underline{10}$-part of χ_α, they saturate the number of degrees of freedom to make the formalism covariant, preserving unitarity, the conformal invariance and the cohomology of the BRST operator.

6. Massless states for open strings

The cohomology of the operator Q_B in [3] — denoted by $H^n(Q_B)$, where n is the ghost number — is a constrained cohomology with the condition (2.9) for pure spinors λ^α. Berkovits argued that $H^1(Q_B)$ must be linear in the field λ^α, because of the nonlinearity of the action of the Lorentz generators in the fields λ_{ab}. It yields the spectrum of the superstring for SYM [22] (or SUGRA for closed strings), but for massive modes, an explicit parametrization of λ^α seems to be needed.

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In our approach based on the Kac-Moody algebra (2.23), we must find the appropriate constraints on the kinematics and the dynamics of physical states. We need therefore both a kinematical constraint which relates the curvatures to the connections (supermultiplets of $N=1$ superspace $D=(9,1)$ in the case of open strings [22]) and a dynamical constraint which imposes the correct equations of motion on the connections (for example the super Yang-Mills equations of motion in $D=(9,1)$ [22]).

In the conventional RNS formalism [9, 23], the dynamical properties of the physical states are encoded in the BRST symmetry which implements, at the quantum level, the super-Virasoro algebra of the spinning string, while the supersymmetry multiplets are selected by means of the GSO projection on the BRST cohomology. The BRST cohomology describes the possible deformations of the super-Virasoro algebra in terms of unintegrated vertex operators $\mathcal{U}^{(1)}$ with ghost number $+1$ or, equivalently, in terms of integrated vertex operators $\oint dz \mathcal{V}_z^{(0)}$. They are related by the equation

$$\left[Q, \mathcal{V}_z^{(0)}\right] = \partial_z \mathcal{U}^{(1)}\,, \tag{6.1}$$

where $\mathcal{V}_z^{(0)}$ has zero ghost number. This equation implies that $\oint dz \mathcal{V}_z^{(0)}$ is BRST invariant.

In our case the BRST algebra encodes the Kac-Moody algebra and the corresponding BRST cohomology should contain all the possible deformations of the underlying algebra. In the following, we will show that indeed the ghost number one BRST cohomology of Q, represented here by the unintegrated vertex $\mathcal{U}^{(1)}$ with ghost number $+1$, describes all the admissible deformations. The constraints on the deformations are obtained as Bianchi identities on the curvatures of the gauge superpotentials $A_\alpha(x,\theta), A_m(x,\theta)$ and of the field strenght A^α. This analysis does not lead to the field equations which are implemented by requiring that the possible deformations preserve the energy-momentum tensor T in equation (4.1).

First, we discuss the BRST cohomology and then we discuss the deformations which preserve the energy-momentum tensor.

Consider the unintegrated vertex $\mathcal{U}^{(1)}$ and the computation of the cohomology for the operator Q. The physical space of vertex operators is contained in the cohomology of Q at ghost number 1, which means that $\mathcal{U}^{(1)}$ satisfies

$$\left[Q, \mathcal{U}^{(1)}\right] = 0\,, \qquad \mathcal{U}^{(1)} \neq [Q, \Omega]\,, \tag{6.2}$$

where Ω is a ghost number zero superfield. Notice that $\mathcal{U}^{(1)}$ could depend on the auxiliary fields η^m and ω_{zm}. Nevertheless, since η^m and ω_{zm}, as well as the current $Q_\eta \equiv \oint dz\, \eta^m \omega_{zm}$, are trivial under the BRST transformations, we expect that the physical observables are independent of them. The requirement that the physical vertex $\mathcal{U}^{(1)}$ does not contain η^m can be written as

$$\oint \eta^m \omega_{zm} \mathcal{U}^{(1)} = 0\,. \tag{6.3}$$

Since $\mathcal{U}^{(1)}$ is a scalar, it can not depend on ω_z^m.

To solve (6.2), we decompose $\mathcal{U}^{(1)}$ as

$$\begin{aligned}\mathcal{U}^{(1)} &= \lambda^\alpha A_\alpha + \xi^m A_m + \chi_\alpha A^\alpha + \\ &+ b\left(\lambda^\alpha\lambda^\beta F_{\alpha\beta} + \lambda^\alpha\xi^m F_{\alpha m} + \xi^m\xi^n F_{mn} + \lambda^\alpha\chi_\beta F_\alpha{}^\beta + \chi_\alpha\xi^m F^\alpha{}_m + \chi_\alpha\chi_\beta F^{\alpha\beta}\right),\end{aligned} \tag{6.4}$$

where $A_\alpha, \ldots, F^{\alpha\beta}$ are arbitrary superfields of x_m, θ^α. Terms with derivative $\partial_z x^m$, $\partial_z\theta^\alpha$ and $d_{z\alpha}$ cannot be present since $\mathcal{U}^{(1)}$ is a worldsheet scalar. Perhaps one may construct a composite field C^z (after suitable bosonization and fermionization to allow contravariant vectors) but the cohomology derived from (6.4) should not change. Note that, because b has ghost number -1 and $b(z)b(w) \sim 0$, we can only include terms which are at most bilinear in ξ^m, χ_α and λ^α. Acting on $\mathcal{U}^{(1)}$ with the BRST charge Q (given in (2.27)), and collecting the independent contributions, we find the following equations

$$\begin{aligned}
\lambda\lambda: &\quad D_{(\alpha}A_{\beta)} - \frac{1}{2}\gamma^m_{\alpha\beta}A_m + F_{\alpha\beta} = 0\,,\\
\lambda\xi: &\quad \partial_m A_\alpha - D_\alpha A_m + \gamma_{m\alpha\beta}A^\beta + F_{\alpha m} = 0\,,\\
\xi\xi: &\quad \partial_{[m}A_{n]} + F_{mn} = 0\,,\\
\lambda\chi: &\quad D_\beta A^\alpha + F_\beta{}^\alpha = 0\,,\\
\xi\chi: &\quad \partial_m A^\alpha + F^\alpha{}_m = 0\,,\\
\chi\chi: &\quad F^{\alpha\beta} = 0\,,
\end{aligned} \tag{6.5}$$

where $D_\alpha \equiv \partial/\partial\theta^\alpha + (1/2)\theta^\beta\gamma^m_{\alpha\beta}\partial/\partial x_m$. The normalization is chosen such that $D_\alpha D_\beta + D_\beta D_\alpha = \gamma^m_{\alpha\beta}\partial_m$. We define $D_{(\alpha}A_{\beta)} = (1/2)(D_\alpha A_\beta + D_\beta A_\alpha)$ and $\partial_{[m}A_{n]} = (1/2)(\partial_m A_n - \partial_n A_m)$. From the terms containing b, we get

$$\begin{aligned}
\lambda\lambda\lambda: &\quad D_{(\alpha}F_{\beta\gamma)} - \frac{1}{2}\gamma^m_{(\alpha\beta}F_{\gamma)m} = 0\,,\\
\lambda\lambda\xi: &\quad \partial_m F_{\alpha\beta} - D_{(\alpha}F_{\beta)m} - \gamma^m_{\alpha\beta}F_{mn} + \gamma_{m\gamma(\alpha}F_{\beta)}{}^\gamma = 0\,,\\
\lambda\xi\xi: &\quad \partial_{[m}F_{\alpha|n]} + D_\alpha F_{mn} - \gamma_{[m|\alpha\beta}F^\beta{}_{n]} = 0\,,\\
\lambda\lambda\chi: &\quad D_{(\alpha}F_{\beta)}{}^\gamma - \frac{1}{2}\gamma^m_{\alpha\beta}F^\gamma{}_m = 0\,,\\
\lambda\xi\chi: &\quad \partial_m F_\alpha{}^\beta - D_\alpha F^\beta{}_m + 2\gamma_{m\alpha\gamma}F^{\beta\gamma} = 0\,,\\
\xi\xi\chi: &\quad -\partial_{[m}F^\alpha{}_{n]} = 0\,,\\
\lambda\chi\chi: &\quad D_\alpha F^{\beta\delta} = 0\,,\\
\xi\chi\chi: &\quad \partial_m F^{\alpha\beta} = 0\,,
\end{aligned} \tag{6.6}$$

These equations are written in terms of superfields, and therefore, supersymmetry is manifest. Equations (6.6) correspond to Bianchi identities for the curvature defined in (6.5), so they are automatically satisfied when the curvatures are expressed in terms of the poten-

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tials as in (6.5). Notice that in $D=(9,1)$-superspace, imposing $F_{\alpha\beta}=F_{m\alpha}=0$ one gets the equations [25]

$$
\begin{aligned}
\gamma^{\alpha\beta}_{[mnpqr]} D_\alpha A_\beta &= 0\,, \\
A_m &= \frac{1}{8}\gamma^{\alpha\beta}_m D_\alpha A_\beta\,, \\
A^\alpha &= \frac{1}{10}\gamma^{m\alpha\beta}\left(D_\beta A_m - \partial_m A_\beta\right)\,.
\end{aligned} \tag{6.7}
$$

These equations imply this other system of equations

$$
\begin{aligned}
D_\alpha A_m - \partial_m A_\alpha + \gamma_{m\alpha\beta}A^\beta &= 0\,, \\
D_\alpha A^\beta &= \frac{1}{4}\gamma^{mn}_{\alpha}{}^{\beta} F_{mn}\,, \\
D_\alpha F_{mn} &= (\gamma_{m\alpha\beta}\partial_n - \gamma_{n\alpha\beta}\partial_m)A^\beta\,.
\end{aligned} \tag{6.8}
$$

The latter, together with the Bianchi identities (6.6), imply the equations of motion for linearized super Yang-Mills [25]

$$
\gamma^m_{\alpha\beta}\partial_m A^\beta = 0\,, \qquad \partial^m F_{mn} = 0\,. \tag{6.9}
$$

In ref. [26] it is shown (in the Wess-Zumino gauge) that (6.9) implies (6.8) and (6.8 implies (6.7). Hence, all three set of equations of motion are equivalent.

The potentials A_α, A_m of the gauge connection and their corresponding curvatures $F_{\alpha\beta}, \dots, F^\alpha{}_m$ parametrize all the possible deformation of the Kac-Moody algebra. Indeed, if we define the new BRST $Q_{\mathcal{U}} = Q + \oint \mathcal{U}^{(1)}$, the nilpotency of the new BRST charge implies (up to terms quadratic in the vertex $\mathcal{U}^{(1)}$) equation (6.2), namely $\left[Q, \oint \mathcal{U}^{(1)}\right] = 0$. The new BRST charge is given by

$$
\begin{aligned}
Q_{\mathcal{U}} = \oint dz \Big[& \lambda^\alpha\left(d_{z\alpha} + A_\alpha\right) - \xi^m\left(\Pi_{zm} - A_m\right) - \chi_\alpha\left(\partial\theta^\alpha - A_\alpha\right) - \\
& - \xi^m \kappa^\alpha_z \gamma_{m\alpha\beta}\lambda^\beta \frac{1}{2}\lambda^\alpha\gamma^m_{\alpha\beta} - \lambda^\beta\beta_{zm} + \\
& + c_z - \frac{1}{2}b\left(\xi^m\partial_z\xi_m - \frac{3}{2}\chi_\alpha\partial_z\lambda^\alpha + \frac{1}{2}\partial_z\chi_\alpha\lambda^\alpha\right) - \frac{1}{2}\partial\left(b\,\chi_\alpha\lambda^\alpha\right) + \\
& + b\Big(\lambda^\alpha\lambda^\beta F_{\alpha\beta} + \lambda^\alpha\xi^m F_{\alpha m} + \xi^m\xi^n F_{mn} + \lambda^\alpha\chi_\beta F_\alpha{}^\beta + \chi_\alpha\,\xi^m F^\alpha{}_m + \\
& \quad + \chi_\alpha\chi_\beta F^{\alpha\beta}\Big)\Big]\,.
\end{aligned} \tag{6.10}
$$

where the Kac-Moody generators $\Pi^m_z, \partial_z\theta^\alpha, d_{z\alpha}$ are shifted by the gauge potentials A_m, A_α and A^α.

In the same way, the energy momentum tensor $T_{zz}(z)$ in equations (4.1) is modified into a new tensor $T^A_{zz}(z) = T_{zz}(z) + \mathcal{V}_z(z)$ where $\mathcal{V}_z(z)$ satisfies the equation $[Q, \mathcal{V}^{(0)}_z] = \partial_z\mathcal{U}^{(1)}$. Finally, by requiring that this $T^A_{zz}(z)$ satisfies the usual OPE, namely

$$
T^A_{zz}(z)T^A_{ww}(w) \sim \frac{2T^A_{ww}(w)}{(z-w)^2} + \frac{\partial_w T^A_{ww}(w)}{(z-w)} + \mathcal{O}(A^2)\,, \tag{6.11}
$$

we find the constraints on the gauge potentials and on the field strenghts. The double poles yield

$$\begin{aligned} \partial^2 A_\alpha + 2\gamma^m_{\alpha\beta}\partial_m A^\beta + D_\alpha D_\beta A^\beta - D_\alpha \partial_m A^m &= 0\,, \\ \partial^2 A^\alpha &= 0\,, \\ \partial^2 A_m - \partial_m \partial_n A^n + \partial_m D_\alpha A^\alpha &= 0\,, \end{aligned} \tag{6.12}$$

and the simple poles give further equations

$$\begin{aligned} D_\alpha\left(\partial^2 A_m - \partial_m \partial_n A^n\right) + \gamma^m_{\alpha\beta}\partial_m A^b + \frac{1}{2}\partial_m \partial^2 A_\alpha + \partial_m D_\alpha D_\beta A^\beta &= 0\,, \\ \partial^2 A_\alpha + D_\alpha D_\beta A^\beta - D_\alpha \partial_n A^n &= 0\,. \end{aligned} \tag{6.13}$$

From these equations, we immediately get the equations of motion and the gauge-fixing

$$\begin{aligned} \partial^2 A_\alpha &= 0\,, \qquad & \gamma^m_{\alpha\beta}\partial_m A^\beta &= 0\,, \qquad & \partial^2 A^\alpha &= 0\,, \\ \partial^2 A_m &= 0\,, & \partial_n A^n &= D_\alpha A^\alpha + f\,. \end{aligned} \tag{6.14}$$

Here f is a constant. These correspond to SYM equations of motion. To see this, it is convenient to expand the superfields A_m and A^α to first power of θ^a in terms of gauge field $a_m(x)$ and of gaugino $u^\alpha(x)$ (in the Wess-Zumino gauge $\theta^\alpha A_\alpha = 0$ [27])

$$\begin{aligned} A_m(x,\theta) &= a_m(x) + \theta^a \gamma_{m\alpha\beta} u^\beta(x) + \cdots\,, \\ A^\alpha(x,\theta) &= u^\alpha(x) + \gamma^{mn,\alpha}_\beta \theta^\beta \partial_m a_n(x) + \cdots\,, \end{aligned} \tag{6.15}$$

where the coefficients in front of the θ-terms are fixed by supersymmetry. The equation $\partial^2 A_m = 0$ implies that $\partial^2 a_m(x) = 0$, while $\gamma^m_{\alpha\beta}\partial_m A^\beta = 0$ implies that $\gamma^m_{\alpha\beta}\partial_m u^\beta(x) = 0$ at zero order in θ. The latter coincides with the gaugino equation of motion. However, we still have a condition at the same order by means of $\partial_n A^n = D_\alpha A^\alpha$ (where the constant f is set to zero for convenience). By inserting the expansions (6.15), we deduce $\partial_m a^m = 0$, hence the gauge field a_m satisfies the usual Maxwell equation of motion in the Landau gauge. As shown in [26] these equations of motion, with the Bianchi identites (6.6) imply the constraints $F_{\alpha\beta} = F_{\alpha m} = 0$.

Notice that $F_{\alpha\beta} = F_{\alpha m} = 0$ parametrize a particular set of all the possible deformations of the Kac-Moody algebra. Namely, those deformations which do not modify the non-abelian Lie algebra of zero modes $\oint dz\, \Pi^m_z$ and $\oint dz\, d_{z\alpha}$. Essentially, this means that among all possible deformations of the Kac-Moody algebra, the physical observables are those which do not modify the non-abelian part of the algebra. Clearly, the abelian part of the algebra due to $\Pi\,\Pi \sim z^{-2}\,, \partial\theta\,\partial\theta \sim 0\,, \Pi\,\partial\theta \sim 0$ and $d\,\partial\theta \sim z^{-2}$ is deformed by the curvatures $F_{mn}, {F_m}^\alpha$ and ${F_\alpha}^\beta$. Finally, we have to point out that the introduction of the $b - c_z$ system to promote the central charge of the Kac-Moody algebra (2.23) to a generator renders the BRST cohomology trivial (as pointed out in [13]), however this parametrizes correctly all the possible deformations of the algebra. It is only the requirement on the deformed energy-momentum tensor which implies the correct equations of motion.

Using the method discussed in [26] to eliminate the auxiliary fields from the superfield A_α, one arrives at the expressions [27]

$$\begin{aligned} A_\alpha &= (\gamma^m\theta)_\alpha \left[a_m - \frac{2}{3}(\theta\gamma_m u) - \frac{1}{8}(\theta\gamma_m\gamma^{nr}\theta)f_{nr} + \cdots\right], \\ A_m &= a_m - (\theta\gamma_m u) - \frac{1}{4}(\theta\gamma_m\gamma^{nr}\theta)(f_{nr} + \frac{2}{3}\gamma_{[n}\partial_{r]}u) + \cdots, \\ A^\alpha &= u^\alpha + \frac{1}{2}(\gamma^{mn}\theta)^\alpha(f_{mn} - \theta\gamma_{[m}\partial_{n]}u) + \cdots. \end{aligned} \tag{6.16}$$

The superfields in (6.16) are written exclusively in terms of the physical gauge field a_m, its field strength f_{mn}, and the gaugino u^α; all auxiliary fields have been eliminated. Moreover, the gauge-fixing condition $\theta^\alpha A_\alpha = 0$ is automatically satisfied.

As a by-product of (6.11) one can also determine the the complete $\mathcal{V}_z^{(0)}$. .It is obtained from the unintegrated vertex U(1) by solving the equation $[Q, \mathcal{V}_z^{(0)}] = \partial_z\mathcal{U}^{(1)}$. The result reads

$$\begin{aligned} \mathcal{V}_z^{(0)} &= \Pi_z^m A_m + \partial_z\theta^\alpha A_\alpha + d_{z\alpha}A^\alpha + \\ &+ 2\beta_z^m\xi^n F_{mn} + \beta_{z\alpha}\lambda^\beta D_\beta A^\alpha + \kappa_z^\alpha\chi_\beta D_\alpha A^\beta + \\ &+ (\beta_z^m\chi_\alpha - \beta_{z\alpha}\xi^m)\,\partial_m A^\alpha + \\ &+ b\left[\beta_z^m\left(2\xi^m\xi^r\partial_r F_{mn} - \xi^n\chi_\alpha\partial_m\partial_n A^\alpha - \lambda^\beta\chi_\alpha\partial_m D_\beta A^\alpha\right) - \right. \\ &\qquad \left. + \kappa_z^\alpha\left(\xi^m\xi^n\gamma_{\alpha\beta[n}\partial_{m]}\partial_n A^\beta + \xi^n\chi_\beta\partial_n D_\alpha A^\beta - \lambda^\alpha\chi_\gamma D_\alpha D_\beta A^\gamma\right)\right] + \\ &+ b\,\partial_z b\left[\frac{1}{4}\xi^m\chi_\alpha\partial_m A^m - \frac{3}{2}\chi_\alpha\lambda^\beta D_\beta A^\alpha\right]. \end{aligned} \tag{6.17}$$

In Berkovits' integrated vertex operator [1] three types of terms appear: one is due to Green-Schwarz, one due to Siegel [10] and the last one introduced by him to covariantize the previous terms under the BRST symmetry. The last term $N^{mn}F_{mn}$ contains the Lorentz generator N^{mn} for the pure spinors. In our approach we find that, in addition to $N^{mn}F_{mn}$, we have to add also the contributions for the other ghosts. The reason is clear: we must rotate all the extra ghost as well, in addition to λ. The last terms in (6.17) proportional to b and $b\,\partial b$ are believed to irrelevant for the amplitude computations.

7. Comments

We end with some comments.

1. Our approach is based on a BRST charge for a Kac-Moody algebra, and it is not, in first instance, of the usual form $\oint dz c^z T_{zz} + \cdots$. The generators of the Kac-Moody algebra yield spacetime symmetries instead of worldsheet symmetries. However, by introducing a ghost field c^z and replacing the ghost fields ξ^m, λ^α and χ_α by $c^z\Pi_z^m, c^z\partial_z\theta^\alpha$ and $c^z d_{z\alpha}$ one recovers the familiar form $Q = \oint c^z T_{zz} + \cdots$.

2. The field c^z plays the role of the diffeomorphism ghost, but in our formalism it should be a composite field. It is under construction. With this c^z one should be able to

construct unintegrated vertex operators and prove the equivalence with the RNS formulation.

3. In [28] an action was constructed for the superstring using Maurer-Cartan equations for the Lie algebra containing $\Pi, \partial\theta, d_\alpha$ and a central charge. The complete action includes a WZNW term. By replacing the exterior derivative d by $Q + d$, one can construct simultaneously the action of [28] and our BRST charge and we can prove the equivalence with the conventional Green-Schwarz formalism. It turns out that the anomaly term in the BRST charge (the term proportional to b) corresponds to the WZNW term in the action. Our action, which did not (yet) include a WZNW term, is separately invariant under our Q as we have shown. This is possible when the WZNW term is also separately invariant under our BRST charge.

4. We have found a field $B_{zz}(z)$ which yields the integrated vertex with $\mathcal{V}_z^{(0)}$ from the unintegrated vertex $\mathcal{U}^{(1)}$. In this construction we found other candidates for $B_{zz}(z)$ which are related by Q-exact terms. Specifically, B^I in (4.10), B^{II} in (4.12), B^{III} in (4.14), and B^{IV} in (4.16) are related as follows
$$B^I - B^J = Q\left[b\left(B^I - B^J\right)\right] . \tag{7.1}$$

5. It might seem that our BRST current is trivial because it can be produced by a similarity transformation of c_z:
$$e^{-\oint(1/2)b\tilde{j}_z^B} c_z e^{\oint(1/2)b\tilde{j}_z^B} = c_z + \tilde{j}_z^B - \oint b\tilde{j}_z^B\tilde{j}_z^B . \tag{7.2}$$
The last term is the anomaly in the BRST charge and the right-hand side is indeed our BRST current j_z^B. However, we also must make this similarity transformation on $\oint B_{zz}dz$ and this does not yield a trivial result. These issues are intimately related to the notion of *big picture* (that is the Hilbert space of the RNS string which contains the zero mode of ξ, where ξ is defined by the bosonization of RNS superghosts β, γ in the usual way). In the RNS case, we can split the BRST charge Q into $Q_0 + Q_1$ where $Q_1 = \oint(1/2)b\gamma^2 = \oint b\eta\,\partial\eta e^{-2\phi}$. A similarity transformation of Q_1 with e^{-xQ_0}, where $x = c\xi\,\partial\xi e^{-2\phi}$, produces Q. Note that since ξ_0 is present, the charge Q is equivalent to the trivial charge Q_1 in the big picture. Also here, one should transform $\oint b_{zz}dz$. Of course, the BRST charge in the RNS formalism is not trivial.

6. In the second section, a new BRST charge is constructed for the superstring. The main idea is to extend the Berkovits' pure spinor formulation by relaxing the pure spinor constraint. After the inclusion of many new ghost terms the final BRST charge (1.2) is
$$Q = \oint(C^M W_M - \frac{1}{2}f^M_{NP}B_M C^N C^P) ,$$
where C^M, B_M are the ghosts and antighosts and W_M are the generators of the space-time Kac-Moody algebra $\Pi^m, d_\alpha, \partial\theta^\alpha$. The physical meaning of this procedure is that physical states are not merely BRST invariant ("gauge-invariant" at the quantum

level), but only invariant up to certain gauge transformations. To avoid obtaining a trivial cohomology, one needs to impose further conditions beyond $Q|\Psi\rangle = 0$ and $|\Psi\rangle \neq Q|\Phi\rangle$. We imposed a condition on the deformed energy-momentum tensor T_{zz} and we also speculated that a composite B_{zz} field might provide equivalent conditions. However, we have now replaced all these proposals by one single requirement based on a new grading [29]. The latter changes the BRST cohomology into an equivariant cohomology, but the results for the spectrum derived in this paper remain unchanged.

7. The BRST operator contains three leading terms $\oint \lambda^\alpha d_\alpha - \xi^m \Pi_m + \chi_\alpha \partial\theta^\alpha$. The physical meaning of the these terms in the context of equivariant cohomology is the following: i) the term with $\partial\theta^\alpha$ states that physical states do not depend on yet another coordinate ϕ_α, first introduced by W. Siegel [10] and also by him eliminated; ii) the contribution with d_α turns the free-field model into the Green-Schwarz interacting theory. This is a subtle matter which is discussed elsewhere [24]. iii) the constraint with Π_m generates the translations of x^m along the vector $\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta$. In Berkovits' approach these transformations vanish due to the pure-spinor constraint, but in our approach $\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta$ is non-vanishing, and for that reason we must impose the constraint with Π_m to achieve the same goal. In terms of equivariant cohomology this means that the we are studying the action of BRST transformations modulo the translations generated by Π_m. The localization procedure entailed by the equivariant cohomology reproduces the pure-spinor constraints.

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A. Antifields

We will elsewhere present the BV formulation of our results. We mention here the first steps.

We add all the possible antifields $x^*_m, \theta^{*,\alpha}, \lambda^{*,\alpha}$, d^*_α, ξ^*_m and χ^*_α (with ghost numbers $-1,-1,-2,-1,-2,-2$; notice that d^*_α is an antiholomorphic vector $d^*_{\bar{z},\alpha}$) and construct the antifield-dependent part of the action from the variation of $x^m, \theta^\alpha, \lambda^\alpha, d_\alpha, \xi^m$ and χ_α

$$\begin{aligned} S_s = \int d^2z \Big[& x^*_m (\xi^m + \frac{1}{2}\lambda^\alpha \gamma^m_{\alpha\beta}\theta^\beta) + (-\Pi_m \lambda^\alpha \gamma^m_{\alpha\beta} - \partial\theta^\alpha \gamma^m_{\alpha\beta}\xi_m + \partial\chi_\beta)\, d^{*\beta} + \\ & + \lambda^\alpha \theta^*_\alpha - \frac{1}{2}\lambda^\alpha \gamma^m_{\alpha\beta}\lambda^\beta \xi^*_m + \xi_m \lambda^\alpha \gamma^m_{\alpha\beta} \chi^{*\beta} \Big] \,. \end{aligned} \tag{A.1}$$

The BRST symmetry is compatible with the supersymmetry if all the fields ξ^m, λ^α and χ_α are susy invariant. Furthermore, the action S_s is supersymmetric if we define

$$[Q_\epsilon, \theta^*_\alpha] = +\frac{1}{2} x^*_m \gamma^m_{\alpha\beta}\epsilon^\beta \,, \qquad [Q_\epsilon, x^*_m] = 0 \,, \tag{A.2}$$

all the other susy transformation rules for the antifields being zero. Moreover, the combination of antifields $\theta^*_\alpha + (1/2) x^*_m \gamma^m_{\alpha\beta}\theta^\beta$ is supersymmetric.

On the other hand, by differentiating the action S_s in (A.1) with respect to fields, we derive the BRST variations of the antifields

$$
\begin{aligned}
\{Q'_B, x^*_m\} &= \partial\Big(\lambda^\alpha \gamma_{m\alpha\beta} d^{*\beta}\Big)\,,\\
[Q'_B, \xi^*_m] &= -x^*_m + \partial\theta^\alpha\gamma_{m\alpha\beta}d^{*\beta} + \lambda^\alpha\gamma_{m\alpha\beta}\chi^{*\beta}\,,\\
[Q'_B, \theta^*_\alpha] &= -\frac{1}{2}x^*_m\gamma^m_{\alpha\beta}\lambda^\beta - \partial\Big(\xi_m\gamma^m_{\alpha\beta}d^{*\beta}\Big) - \gamma^m_{\alpha\beta}\partial\theta^\beta\lambda^\delta\gamma_{m\delta\gamma}d^{*\gamma} - \frac{1}{2}\theta^\beta\partial\Big(\lambda^\delta\gamma_{m\delta\gamma}d^{*\gamma}\Big)\,,\\
\{Q'_B, \lambda^*_\alpha\} &= \theta^*_\alpha + \frac{1}{2}x^*_m\gamma^m_{\alpha\beta}\theta^\beta - \Pi_m\gamma^m_{\alpha\beta}d^{*\beta} - \xi^*_m\gamma^m_{\alpha\beta}\lambda^\beta + \xi_m\gamma^m_{\alpha\beta}\chi^{*\beta}\,,\\
\{Q'_B, \chi^{*\alpha}\} &= -\partial d^{*\alpha}\,,\\
[Q'_B, d^{*\alpha}] &= 0\,.
\end{aligned}
\tag{A.3}
$$

There are duality relations among fields and antifields, for example the linear shift of x^m by means of the vector ξ^m corresponds to the linear shift of ξ^*_m by means of x^*_m. In the same way, the variation of d_α contains the holomorphic derivative of χ_α, which is dual to the variation of $\chi^{*\alpha}$ into $-\partial_z d^{*,\alpha}$. This duality between fields and antifields is typical for topological field theories quantized with the BV formalism. To reveal such topological aspects, it is convenient to introduce new variables

$$
\begin{aligned}
\widetilde{\xi}^m &= \xi^m + \frac{1}{2}\theta^\alpha\gamma^m_{\alpha\beta}\lambda^\beta\,,\\
\widetilde{d}_{z\alpha} &= d_{z\alpha} + \partial_z x^m\gamma_{m\alpha\beta}\theta^\beta + \frac{1}{6}\gamma^m_{\alpha\beta}\theta^\beta\theta^\gamma\gamma_{m\gamma\delta}\partial_z\theta^\delta\,,\\
\widetilde{\chi}_\alpha &= \chi_\alpha + \widetilde{\xi}^m\gamma_{m\alpha\beta}\theta^\beta + \frac{1}{6}\gamma^m_{\alpha\beta}\theta^\beta\theta^\gamma\gamma_{m\gamma\delta}\lambda^\beta\,.
\end{aligned}
\tag{A.4}
$$

Inserting the expression for $d_{z\alpha}$ into (A.4), we find that $\widetilde{d}_{z\alpha} = j^\epsilon_{z\alpha}$ where $j^\epsilon_{z\alpha}$ is the current for the supersymmetric charge given in (2.5). In terms of the new variables the BRST transformations simplify to

$$
\begin{aligned}
[Q, x^m] &= \widetilde{\xi}^m\,, & \{Q, \widetilde{\xi}^m\} &= 0\,,\\
\{Q, \theta^\alpha\} &= \lambda^\alpha\,, & [Q, \lambda^\alpha] &= 0\,,\\
\{Q, j^\epsilon_{z\alpha}\} &= \partial_z\widetilde{\chi}_\alpha\,, & [Q, \widetilde{\chi}_\alpha] &= 0\,,
\end{aligned}
\tag{A.5}
$$

which clearly show the topological nature of the BRST symmetry generated by Q. Notice that although the BRST symmetry is very simple in terms of the new variables, the variables $\widetilde{\xi}_m, \widetilde{d}_{z\alpha}$ and $\widetilde{\chi}_\alpha$ are no longer susy invariant. In terms of the old variables the BRST symmetry is rather complicated, but Π_m, d_α and all the ghost fields are supersymmetric (clearly x^m and θ^α are not supersymmetric being the coordinates of the superspace).

Due to the simple structure of (A.5), we can write the associated BRST charge as

$$
\begin{aligned}
Q &= \oint dz j^B_z\,,\\
j^B_z &\equiv \lambda^\alpha\left(j^\epsilon_{z\alpha} + \mathcal{J}^\epsilon_{z\alpha}\right) - \widetilde{\xi}^m\,\partial_z x_m + \partial_z\hat{\chi}_\alpha\,\theta^\alpha\,,\\
\mathcal{J}^\epsilon_{z\alpha} &\equiv \frac{1}{2}\widetilde{\beta}_{zm}\gamma^m_{\alpha\beta}\lambda^\beta - \widetilde{\xi}_m\gamma^m_{\alpha\beta}\widetilde{\kappa}^\beta_z\,,
\end{aligned}
\tag{A.6}
$$

where $\mathcal{J}^{\epsilon}_{z\alpha}$ generates the susy transformations on the ghost fields. Here, $\widetilde{\beta}^m_z$ and $\widetilde{\kappa}_{z\alpha}$ are the antighost for $\widetilde{\xi}^m$ and for χ_α, respectively. Notice that according to this BRST charge, the ghosts $\widetilde{\xi}^m, \widetilde{\chi}_\alpha$ and λ^α are associated with the generators of the translations and of the supersymmetry transformations.

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